



Student Name/Number: \_\_\_\_\_

**2016**

## **Year 12 Mathematics**

Trial Examination

Teacher Setting Paper: Miss K Cole  
Head of Department: Mrs M Hill

### **General Instructions**

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen  
(Black pen is preferred)
- Board approved calculator may be used
- Write your answers for Section I on the multiple answer sheet provided
- Write your student number only on the front of each booklet
- A formula reference sheet is provided at the back of this paper
- In Questions 11- 16, show relevant mathematical reasoning and/or calculations.

### **Total marks – 100**

#### **Section I – Multiple Choice**

**10 marks**

Attempt Questions 1-10

Allow 15 minutes for this section

#### **Section II – Extended Response**

**90 marks**

Attempt questions 11 - 16

Allow 2 hour and 45 minutes for this section

*This examination paper does not necessarily reflect the content or format of the Higher School Certificate Examination in this subject.*

Student Name/Number: \_\_\_\_\_

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**Section I****10 marks****Attempt Questions 1 – 10**

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Use the multiple-choice answer sheet for Questions 1 - 10.

**QUESTION 1**What is  $\sqrt{\frac{2.91^{13}}{2.13^{11} \times 1.37^9}}$  correct to 3 significant figures?

- (A) 3.92
- (B) 3.926
- (C) 3.93
- (D) 3.924

**QUESTION 2**What does  $x$  represent if  $\sqrt{75} + \sqrt{108} = x\sqrt{3}$ ?

- (A)  $x = 5$
- (B)  $x = 6$
- (C)  $x = 11$
- (D)  $x = 14$

**QUESTION 3**The equation of the straight line that is perpendicular to line  $4x - 3y + 2 = 0$  and passes through  $(8, -3)$  is:

- (A)  $4x - 3y - 26 = 0$
- (B)  $4x - 3y - 41 = 0$
- (C)  $3x - 4y - 36 = 0$
- (D)  $3x + 4y - 12 = 0$

**QUESTION 4**

What is the amplitude and period of the curve  $y = 3 \cos 2x$ ?

- (A) Amplitude = 3, period =  $2\pi$
- (B) Amplitude = 2, period =  $3\pi$
- (C) Amplitude = 3, period =  $\pi$
- (D) Amplitude = 2, period =  $\frac{3\pi}{2}$

**QUESTION 5**

For what values of  $k$  does the equation  $x^2 - (k + 6)x - 4 = 0$  have no real roots?

- (A)  $k > -10$
- (B)  $k < -2$
- (C)  $k = -10, k = -2$
- (D)  $-10 < k < -2$

**QUESTION 6**

The equation of the tangent to the curve  $y = 6x - \frac{1}{x^2}$  through the point (1, 6) is:

- (A)  $4x - y + 2 = 0$
- (B)  $4x - y - 2 = 0$
- (C)  $8x - y - 2 = 0$
- (D)  $8x - y + 2 = 0$

**QUESTION 7**

Two regular six-sided dice, with the numbers 1 to 6 on their faces, are rolled simultaneously. What is the probability that at least one of them shows a 6?

(A)  $\frac{11}{36}$

(B)  $\frac{1}{3}$

(C)  $\frac{1}{6}$

(D)  $\frac{11}{18}$

**QUESTION 8**

Evaluate  $\sum_{k=1}^{50} 2k + 3$

(A) 5292

(B) 2700

(C) 5400

(D) 2646

**QUESTION 9**

The primitive function of  $\frac{1}{\sqrt{x}}$  is:

(A)  $2\sqrt{x} + C$

(B)  $\frac{\sqrt{x}}{2} + C$

(C)  $-\frac{1}{2\sqrt{x}} + C$

(D)  $-\frac{1}{2\sqrt[3]{x}} + C$

**QUESTION 10**

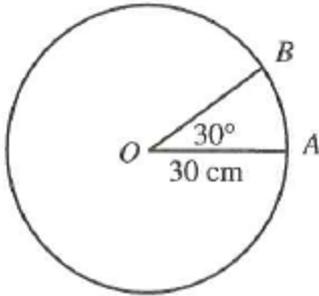
The domain and range of the function  $y = \frac{1}{x-1}$  is:

- (A) Domain:  $x < -1, x > -1$ , Range:  $y < 0, y > 0$
- (B) Domain:  $x < 1, x > 1$ , Range:  $y < 0, y > 0$
- (C) Domain:  $x < 0, x > 0$ , Range:  $y < 0, y > 0$
- (D) Domain: all real  $x$ , Range: all real  $y$

**END OF SECTION I**

**Section II****90 marks****Attempt Questions 11-16**

Write your answers on the Booklets provided

- QUESTION 11** (15 marks) (Start a new booklet) **Marks**
- (a) Write with a rational denominator,  $\frac{1}{\sqrt{6}-2}$  **2**
- (b) Solve  $|4-x|=1$  **2**
- (c)  $O$  is the centre of the circle.
- 
- Find the exact length of the minor arc  $AB$ . **2**
- (d) If  $\alpha$  and  $\beta$  are the roots of  $2x^2 + 3x - 6 = 0$ , what is the value of  $\frac{\alpha\beta}{\alpha + \beta}$ ? **2**
- (e) Solve the simultaneous equations **2**
- $$y = 3x$$
- $$x - 2y = 10$$
- (f) Differentiate:
- (i)  $\frac{1}{3x^3}$  **1**
- (ii)  $5x \sin x$  **2**
- (iii)  $\ln(2x+1)$  **2**

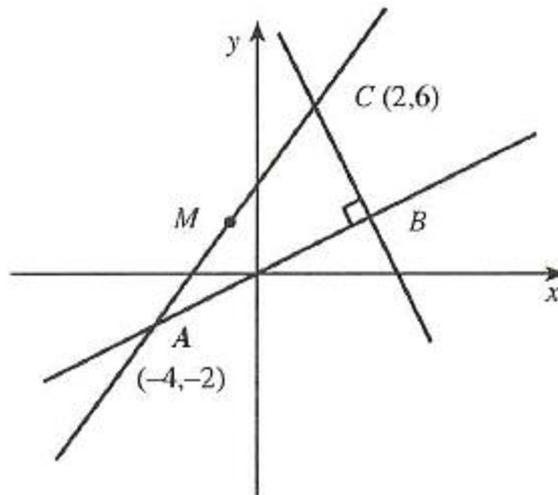
**End of Question 11**

**QUESTION 12** (15 marks)

(Start a new booklet)

**Marks**

- (a) In the following diagram,
- $A(-4, -2)$
- and
- $C(2, 6)$
- are points of intersection.



- (i) Find the equation of the line  $AB$ , given that it passes through the origin. **1**
- (ii) The line  $BC$  is perpendicular to  $AB$ . Show that its equation is  $y = -2x + 10$  **1**
- (iii) Find the length of  $AC$ . **1**
- (iv) Find the coordinates of  $M$ , the midpoint of  $AC$ . **2**
- (v) Given a circle, centre  $M$ , can be drawn to pass through  $A$ ,  $B$  and  $C$ , write down the equation of this circle. **2**
- (b) Show that the normal to the curve  $y = \frac{x^2}{4}$  at the point  $(4, 4)$  has the equation **2**  
 $2y = 12 - x$ .
- (c) An infinite geometric series has a first term of  $-3$ , a common ratio of  $r$ , and a limiting sum of  $4r$ . Find the value(s) of  $r$ . **2**

**Question 12 (continued)**

(d) Find

(i)  $\int e^{3x} dx$  **2**

(ii)  $\int_0^{\frac{2\pi}{3}} \sin \frac{x}{2} dx$  **2**

**End of Question 12**

**QUESTION 13** (15 marks) (Start a new booklet) **Marks**

(a) Consider the curve  $y = 3x^2 - x^3$

(i) Find the stationary points and determine their nature. **3**

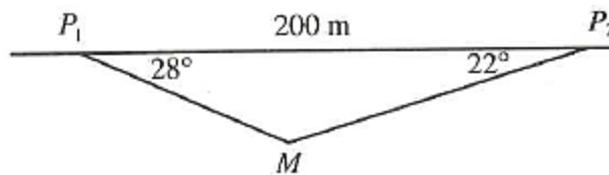
(ii) Sketch the curve. **2**

(b) For the parabola  $(x-2)^2 = 4y$ , find the coordinates of

(i) Its vertex **1**

(ii) Its focus **1**

(c)  $P_1$  and  $P_2$  are points 200 m apart on horizontal ground. Mine shafts are driven from  $P_1$  and  $P_2$  as shown to meet underground at  $M$ . Find, to the nearest metre,

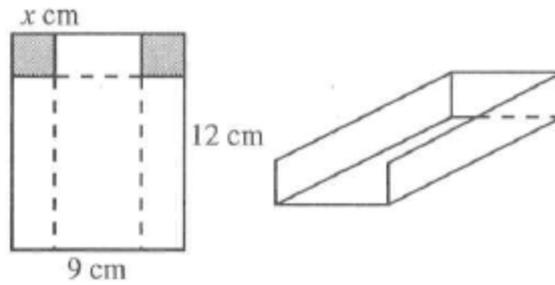


(i) The length of the shaft  $P_1M$  **2**

(ii) The vertical depth  $M$  below the surface. **1**

**Question 13 (continued)**

- (d) A rectangle of cardboard measures 12 cm by 9 cm. From 2 corners, squares of side  $x$  cm are removed, as shown. The remainder is folded along the dotted lines to form a tray.



- (i) Show that the volume,  $V$  cm<sup>3</sup>, of the tray is given by  $V = 2x^3 - 33x^2 + 108x$ . **2**
- (ii) Find the maximum possible volume of the tray. **3**

**End of Question 13**

**QUESTION 14** (15 marks) (Start a new booklet) **Marks**

(a) A cube has three green faces, two white faces, and one red face. If a player throws a green face, they win; if red, they lose; and if white, they may throw again. Megan will throw until she either wins or loses. What is the probability that

(i) Megan wins with her second throw? **2**

ii) Megan wins with either her first, second or third throw? **2**

(b) (i) Copy and complete the table below with decimals correct to three places. **2**

$x$	2	3	4	5
$\ln x$				

(ii) Use the table and the Trapezoidal rule with four function values to find an approximation to  $\int_2^5 \ln x dx$ . **2**

(iii) Show that  $\frac{d}{dx}(x \ln x - x) = \ln x$ . **2**

(iv) Hence, find the exact value of the integral in part (ii), correct to three decimal places. **2**

(c) (i) Sketch a graph of  $y = 9 - x^2$  **1**

(ii) Calculate the area enclosed by this graph and the  $x$  axis. **2**

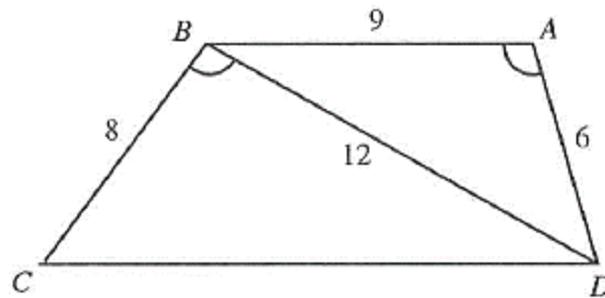
**End of Question 14**

**QUESTION 15** (15 marks)

(Start a new booklet)

**Marks**

- (a) In the diagram,
- $\angle CBD = \angle DAB$

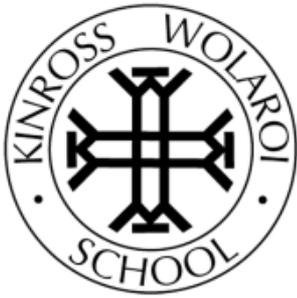


- (i) Prove triangles  $ABD$  and  $BDC$  similar. 2
- (ii) Find the length of  $CD$ . 1
- (iii) Prove that  $AB$  and  $CD$  are parallel. 1
- (b) A father gives his son \$100 on his 15<sup>th</sup> birthday and then on each succeeding birthday he gives him 10% less than the previous one.
- (i) How much does the son receive on his 21<sup>st</sup> birthday? 2
- (ii) Show that the total amount he may receive will not exceed \$1000. 2
- (c) In order to film an outdoor scene, a director has a camera mounted on a trolley which runs on a straight track. Consider the track to be represented by an  $x$  axis, graduated in metres. Initially, filming begins with the camera at  $x = 49$ . After  $t$  minutes, the velocity,  $v$  m/min of the trolley is given by  $v = 4t^3 - 100t$ ,  $t \geq 0$ .
- (i) Find the position  $x$  of the camera as a function of  $t$ . 2
- (ii) Show that the camera passes through the origin twice. 2
- (d) One of the roots of the equation  $2x^2 - 15x + c = 0$  is four times the other.
- (i) Find the roots. 2
- (ii) Find the value of  $c$ . 1

**End of Question 15**

- QUESTION 16** (15 marks) (Start a new booklet) **Marks**
- (a) (i) Show that, if  $y = \frac{\sin x}{\sin x + \cos x}$ , then  $\frac{dy}{dx} = \frac{1}{(\sin x + \cos x)^2}$ . **2**
- (ii) The region under the curve  $y = \frac{1}{\sin x + \cos x}$  above the  $x$  axis, and between  $x = 0$  and  $x = \frac{\pi}{4}$ , makes a revolution about the  $x$  axis. Find the volume of the solid formed. **2**
- (b) The diagram below shows part of the floor plan for a proposed concert hall. The floor narrows from front to back so that each row of seats behind the first has two less seats than the row in front of it. The first row has fifty-seven seats.
- 
- (i) Write an expression for the number of seats in the  $n^{\text{th}}$  row? **2**
- (ii) What is the greatest value  $n$  can take? **1**
- (iii) The hall is planned to seat 720 people. How many rows of seats will there be? **2**
- (c) The mass,  $M$  g, of a radioactive element present in a substance after  $t$  years is given by  $M = M_0 e^{-kt}$ , where  $M_0$  is the initial mass and  $k$  is a constant. The half-life of the element is 100 years.
- (i) Show that  $k = \frac{\ln 2}{100}$  **2**
- (ii) How long will it take for 9 g to reduce to 2 g? **2**
- (iii) What percentage of the original mass will be present after 32 years? **2**

**End of examination**



**2016**

**Year 12 Mathematics**

Trial Examination

**MULTIPLE CHOICE ANSWER SHEET**

For multiple choice questions, choose the best answer A, B, C or D and fill in the correct circle.

1.     A    B    C    D
2.     A    B    C    D
3.     A    B    C    D
4.     A    B    C    D
5.     A    B    C    D
6.     A    B    C    D
7.     A    B    C    D
8.     A    B    C    D
9.     A    B    C    D
10.    A    B    C    D

2016 2 Unit Trial Exam Solutions

1. 3.9262...  
3.93

(C)

2.  $\sqrt{75} + \sqrt{108}$   
 $= 5\sqrt{3} + 6\sqrt{3}$   
 $= 11\sqrt{3}$   
 $x = 11$

(C)

3.  $4x - 3y + 2 = 0$  (8, -3)  
 $m = \frac{4}{3}$   $\perp m = -\frac{3}{4}$

$y + 3 = -\frac{3}{4}(x - 8)$

$4(y + 3) = -3(x - 8)$

$4y + 12 = -3x + 24$

$3x + 4y - 12 = 0$

(D)

4.  $y = 3 \cos 2x$   
amplitude = 3  
period =  $\frac{2\pi}{2} = \pi$

(C)

5.  $x^2 - (k+b)x = -4$   $\Delta < 0$

$[-(k+b)]^2 - 4(1)(4) < 0$

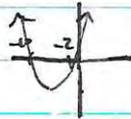
$k^2 + 12k + 36 - 16 < 0$

$k^2 + 12k + 20 < 0$

$(k+2)(k+10) < 0$

$k = -2$   $k = -10$

$-10 < k < -2$



(D)

6.  $y = 6x - \frac{1}{x^2}$  (1, 6)

$y = 6x - x^{-2}$

$y' = 6 + 2x^{-3}$

$m = 6 + 2(1)^{-3}$   
 $= 8$

$y - 6 = 8(x - 1)$

$y - 6 = 8x - 8$

$8x - y - 2 = 0$

(C)

7.  $P(\geq \text{one } 6) = \frac{6+6-1}{36}$   
 $= \frac{11}{36}$

(A)

8.  $S_{50} = \frac{50}{2}(5+103)$   $T_1 = 2(1)+3 = 5$   
 $= 2700$   $T_{50} = 2(50)+3 = 103$

(B)

9.  $x^{-1/2}$   
 $= \frac{x^{1/2}}{1/2} + C$   
 $= 2\sqrt{x} + C$

(A)

10.  $y = \frac{1}{x-1}$   $x-1 \neq 0$   $y \neq 0$   
 $x \neq 1$

Domain =  $x < 1, x > 1$

Range =  $y < 0, y > 0$

(B)

11 a)  $\frac{1}{\sqrt{6}-2} \times \frac{(\sqrt{6}+2)}{\sqrt{6}+2}$  (✓)  
 $= \frac{\sqrt{6}+2}{6-4}$   
 $= \frac{\sqrt{6}+2}{2}$  or  $\frac{\sqrt{6}}{2} + 1$  (✓)

b)  $|4-x|=1$   
 $4-x=1$   $-(4-x)=1$   
 $-x=-3$   $-4+x=1$   
 $x=3$  (✓)  $x=5$  (✓)

c)  $l = r\theta$   $30^\circ = \frac{\pi}{6}$  (✓)  
 $l = 30 \times \frac{\pi}{6}$   
 $l = 5\pi$  cm (✓)

# 2016 2 Unit Mathematics Trial Exam Solutions

②

11. d)  $2x^2 + 3x - 6 = 0$

$$\alpha\beta = \frac{-b}{a} = -\frac{3}{2}$$

✓

$$\alpha + \beta = \frac{-3}{2}$$

$$\frac{\alpha\beta}{\alpha + \beta} = \frac{-3}{-3/2}$$

$$= 2 \quad \text{✓}$$

e) ①  $y = 3x$

②  $x - 2y = 10$

$$x - 2(3x) = 10 \quad \text{sub ① into ②}$$

$$x - 6x = 10$$

$$-5x = 10$$

$$x = -2 \quad \text{✓}$$

$$y = 3(-2)$$

$$y = -6 \quad \text{✓}$$

f) i)  $\frac{1}{3x^3} = \frac{1}{3}x^{-3}$

$$\frac{d}{dx} = \frac{1}{3}x^{-4}$$

$$= -\frac{1}{3}x^{-5} \quad \text{✓}$$

ii)  $5x \sin x$

$$u = 5x \quad v = \sin x$$

$$u' = 5 \quad v' = \cos x$$

$$\frac{d}{dx} = 5x \cos x + 5 \sin x$$

✓

✓

iii)  $\ln(2x+1)$

$$f(x) = 2x+1$$

$$\frac{d}{dx} = \frac{2}{2x+1} \quad \text{✓}$$

$$f'(x) = 2$$

$$= \frac{2}{2x+1} \quad \text{✓}$$

12 a) i)  $(0,0) (-4,-2)$

$$m = \frac{-2}{-4} = \frac{1}{2}$$

$$y = \frac{1}{2}x \quad \text{or} \quad x - 2y = 0 \quad \text{✓}$$

12 a) ii)  $m_{AB} = \frac{1}{2}$

$$m_1 \times m_2 = -1$$

$$m_{BC} = -2$$

$$C(2,6)$$

$$b = -2(2) + b$$

$$6 = -4 + b$$

$$b = 10$$

$$y = -2x + 10 \quad \text{✓}$$

iii)  $d = \sqrt{(6-2)^2 + (2-4)^2}$

$$= \sqrt{64+36}$$

$$= \sqrt{100}$$

$$= 10 \quad \text{✓}$$

iv)  $M = \left( \frac{-4+2}{2}, \frac{-2+6}{2} \right)$

$$M = (-1, 2) \quad \text{✓} \quad \text{✓}$$

v)  $MA = \frac{1}{2} AB$

$$= \frac{1}{2} \times 10$$

$$= 5 \text{ (radius)}$$

$$(x+1)^2 + (y-2)^2 = 25 \quad \text{✓} \quad \text{✓}$$

b)  $y = \frac{x^2}{4} \quad (4,4)$

$$y' = \frac{2x}{4} = \frac{x}{2}$$

$$m = \frac{4}{2} = 2$$

$$l_m = -\frac{1}{2} \quad \text{✓}$$

$$y - 4 = -\frac{1}{2}(x - 4)$$

$$2(y - 4) = -(x - 4)$$

$$2y - 8 = -x + 4$$

$$2y = 12 - x \quad \text{✓}$$

2016 2 Unit Mathematics Trial Exam Solutions <sup>(3)</sup>

12. c)  $a = -3$   $r = r$   $S_{\infty} = 4r$   
 $4r = \frac{-3}{1-r}$   $|r| < 1$

$4r(1-r) = -3$   
 $4r - 4r^2 = -3$   
 $4r^2 - 4r - 3 = 0$   
 $(2r+1)(2r-3) = 0$   
 $r = -\frac{1}{2}$   $r = \frac{3}{2}$

d) i)  $\int e^{3x} dx = \frac{1}{3} e^{3x} + C$

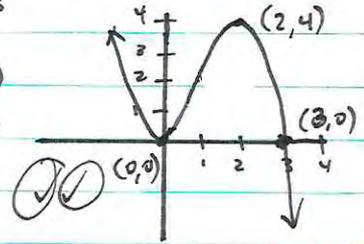
ii)  $\int_0^{\frac{2\pi}{3}} \sin \frac{x}{2} dx$   
 $= \left[ -2 \cos \frac{x}{2} \right]_0^{\frac{2\pi}{3}}$   
 $= -2 \left( \cos \frac{\pi}{3} - \cos 0 \right)$   
 $= -2 \left( \frac{1}{2} - 1 \right)$   
 $= -2 \left( -\frac{1}{2} \right)$   
 $= 1$

13 a) i)  $y = 3x^2 - x^3$   
 $y' = 6x - 3x^2$   
 $0 = 3x(2-x)$   
 $x = 0$   $x = 2$   
 $y = 0$   $y = 3(2)^2 - 2^3$   
 $= 4$

x	-1	0	1	2	3
y'	\	-	/	-	\

(0,0) is minimum  $\checkmark$   $\checkmark$   
 (2,4) is maximum  $\checkmark$   $\checkmark$

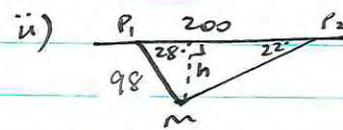
13 a) ii)  $0 = 3x^2 - x^3$   
 $0 = x^2(3-x)$   
 $x = 0$   $x = 3$



b)  $(x-2)^2 = 4y$   
 i) (2,0)  $\checkmark$   
 ii) focal length = 1  
 (2,1)  $\checkmark$

c) i)  $\angle M = 180 - (22 + 28)$   
 $= 130^\circ$

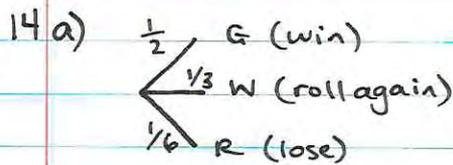
$\frac{P_1 M}{\sin 22^\circ} = \frac{200}{\sin 130^\circ}$   
 $P_1 M = \frac{200 \times \sin 22^\circ}{\sin 130^\circ}$   
 $= 97.8 \dots$   
 $= 98 \text{ m}$



$\sin 28^\circ = \frac{h}{98}$   
 $h = 98 \times \sin 28^\circ$   
 $= 46.008 \dots$   
 $= 46 \text{ m}$

d) i)  $V = x(9-2x)(12-x)$   
 $= (9x - 2x^2)(12-x)$   
 $= 108x - 9x^2 - 24x^2 + 2x^3$   
 $V = 2x^3 - 33x^2 + 108x$

ii)  $V' = 6x^2 - 66x + 108$   
 $0 = 6(x^2 - 11x + 18)$   
 $0 = (x-2)(x-9)$   
 $x = 2$   $x = 9$   
 $V = 2(2)^3 - 33(2)^2 + 108(2)$   
 $= 100 \text{ cm}^3$



i)  $P(WG) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

ii)  $P(G + WG + WWG)$   
 $= \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{2}$   
 $= \frac{1}{2} + \frac{1}{6} + \frac{1}{18}$   
 $= \frac{13}{18}$

b) i)

x	2	3	4	5
lnx	0.693	1.099	1.386	1.609

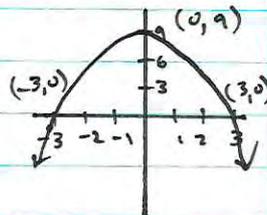
ii)  $\int_2^5 \ln x dx$   
 $\approx \frac{1}{2} (0.693 + 2(1.099 + 1.386) + 1.609)$   
 $\approx 3.636 \text{ units}^2$

iii)  $x \ln x - x$        $u = x$     $v = \ln x$   
 $u' = 1$     $v' = \frac{1}{x}$

$\frac{d}{dx} = \ln x + x \times \frac{1}{x} - 1$   
 $= \ln x + 1 - 1$   
 $= \ln x$

iv)  $\int_2^5 \ln x dx$   
 $= [x \ln x - x]_2^5$   
 $= (5 \ln 5 - 5) - (2 \ln 2 - 2)$   
 $= 3.047 + 0.614$   
 $= 3.661 \text{ units}^3$

c) i)  $y = 9 - x^2$   
 $= (3-x)(3+x)$   
 $x = \pm 3$



H. c) ii)  $2 \int_0^3 (9 - x^2) dx$   
 $= 2 [9x - \frac{x^3}{3}]_0^3$   
 $= 2 ((9(3) - \frac{3^3}{3}) - (9(0) - \frac{0^3}{3}))$   
 $= 2 \times 18$   
 $= 36 \text{ units}^2$

15 a) i) In  $\triangle ABD$  and  $\triangle BDC$   
 $\angle DAB = \angle DBC$  (given)

$\frac{AD}{BC} = \frac{6}{8} = \frac{3}{4}$

$\frac{AB}{BD} = \frac{9}{12} = \frac{3}{4}$

$\therefore \triangle ABD \parallel \triangle BDC$

(2 pairs of corresponding sides in proportion + included angles equal)

ii)  $\frac{12}{CD} = \frac{3}{4}$

$CD = 16$

iii)  $\angle ABD = \angle CDB$

( $\angle$ s in similar  $\triangle$ s are equal)

$\therefore AB \parallel CD$  (alternate  $\angle$ s are equal in parallel lines)

b) i)  $100 + 0.9 \times 100 + 0.9^2 \times 100 + \dots$   
 15th                      16th                      17th

$\sum_{15}^{21} 100(0.9)^{n-1}$                        $21 - 15 + 1 = 7$

$T_7 = 100(0.9)^6$

$= \$53.14$

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15 b) ii)  $S_{\infty} = \frac{100}{1-0.9}$   
 $= 1000$

The limiting sum approaches \$1000 so the total will always be less than \$1000.

c) i)  $v = 4t^3 - 100t$   
 $x = t^4 - 50t^2 + C$   $t=0, x=49$   
 $49 = 0^4 - 50(0)^2 + C$   
 $C = 49$   
 $x = t^4 - 50t^2 + 49$

ii)  $0 = t^4 - 50t^2 + 49$   
 $0 = (t^2 - 1)(t^2 - 49)$   
 $0 = (t+1)(t-1)(t+7)(t-7)$   
 $t = \pm 1 \quad t = \pm 7$   
 since time can't be negative  
 $t = 1 \text{ min} \quad t = 7 \text{ min}$

16 a) i)  $y = \frac{\sin x}{\sin x + \cos x}$   
 $u = \sin x \quad v = \sin x + \cos x$   
 $u' = \cos x \quad v' = \cos x - \sin x$   
 $\frac{dy}{dx} = \frac{\cos(\sin x + \cos x) - \sin x(\cos x - \sin x)}{(\sin x + \cos x)^2}$   
 $= \frac{\sin x \cos x + \cos^2 x - \sin x \cos x + \sin^2 x}{(\sin x + \cos x)^2}$   
 $= \frac{\sin^2 x + \cos^2 x}{(\sin x + \cos x)^2}$   
 $= \frac{1}{(\sin x + \cos x)^2}$

15 d)  $\alpha = 4\beta$

i)  $\alpha + \beta = \frac{15}{2}$

$5\beta = \frac{15}{2}$

$\beta = 1.5$

ii)  $\alpha\beta = \frac{c}{2}$

$9 = \frac{c}{2}$

$c = 18$

16 a) ii)  $V = \pi \int_0^{\pi/4} \left( \frac{1}{\sin x + \cos x} \right)^2 dx$

$= \pi \left[ \frac{\sin x}{\sin x + \cos x} \right]_0^{\pi/4}$

$= \pi \left( \frac{\sin \pi/4}{\sin \pi/4 + \cos \pi/4} - \frac{\sin 0}{\sin 0 + \cos 0} \right)$

$= \pi \left( \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} - \frac{0}{0+1} \right)$

$= \pi \left( \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{2} \right)$

$= \frac{\pi}{2} \text{ units}^2$

b) i)  $a = 57 \quad d = -2$

$T_n = 57 + (n-1)(-2)$   
 $= 59 - 2n$

ii)  $T_n \geq 0$

$59 - 2n \geq 0$

$-2n \geq -59$

$n \leq 29.5$

$n = 29$  is greatest value

iii)  $S_n = 720$

$720 = \frac{n}{2}(2 \times 57 + (n-1)(-2))$

$720 = \frac{n}{2}(116 - 2n)$

$720 = 58n - n^2$

$n^2 - 58n + 720 = 0$

$(n-18)(n-40) = 0$

$n = 18 \quad n = 40$

18 rows

c) i)  $M = M_0 e^{-kt}$

$\frac{1}{2} = e^{-k(100)}$

$\ln \frac{1}{2} = -100k$

$\ln(2)^{-1} = -100k$

$-\ln 2 = -100k$

$k = \frac{\ln 2}{100}$

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(6)

16 c) ii)  $2 = 9 e^{-\frac{\ln 2}{100} t}$

$$\frac{2}{9} = e^{-\frac{\ln 2}{100} t}$$

$$\ln \frac{2}{9} = \frac{-\ln 2}{100} t \quad (\checkmark)$$

$$t = \frac{\ln \frac{2}{9}}{\left(-\frac{\ln 2}{100}\right)}$$

$$t = 216.9925\dots$$

$$t \approx 217 \text{ years} \quad (\checkmark)$$

iii)  $M = 1 e^{-\frac{\ln 2}{100} \times 32} \quad (\checkmark)$

$$= 0.80106$$

$$\approx 80\% \quad (\checkmark)$$